This paper investigates a hybrid methodology that combines fuzzy logic and neural networks, Fuzzy Cognitive Map (FCM), for modeling and controlling Supervisory Control Systems. A mathematical description of Fuzzy Cognitive Maps (FCM) will be presented and new construction methods will be extensively examined. A Fuzzy Cognitive Map will be developed to model and control a process example and the Supervisor-FCM model characteristics will be discussed. There is a going need for more autonomous and intelligent systems, especially in Complex Systems area, the application of Fuzzy Cognitive Maps for modeling the Supervisor may contribute to develop more sophisticated systems.

1. Introduction

Modern systems are characterized as complex systems with high dimension and a variety of variables and factors. For complex dynamical systems, conventional methods have a limited contribution in modeling and controlling such systems and new techniques are required for developing sophisticated systems. New methods have proposed for complex systems that will utilize existence knowledge and human experience and will have learning capabilities and advanced characteristics such as failure detection and identification qualities. In this paper Fuzzy Cognitive Maps (FCM) are proposed for modeling and controlling complex systems. The application of FCM may contribute to the effort for more intelligent control methods and to the development of autonomous systems. A Fuzzy Cognitive Map draws a causal picture to represent the model and the behavior of system. The concepts of an FCM interact according to imprecise rules and the operation of complex systems is simulated.

Fuzzy Cognitive Maps are symbolic representation for the description and modeling of the system. They consist of concepts, that illustrate different aspects in the behavior of the system and these concepts interact with each other showing the dynamics of the system [7]. The human experience and knowledge of the operation of the system is used to develop the Fuzzy Cognitive Map, as a result of the method by which it is constructed, i.e., using human experts that know the operation of system and its behavior in different circumstances. An FCM describes the behavior of a system in terms of concepts; each concept represents a state, a variable, or a characteristic of the system [2]. An FCM illustrates the whole system by a graph showing the cause and effect along concepts, and it is a simple way to describe the system’s behavior in a symbolic manner, exploiting the accumulated knowledge of the system.

Fuzzy Cognitive Map (FCM) have been applied in a variety of scientific areas, FCMs have been used to describe and model the behavior of a system and its application in modeling the supervisor of distributed systems [14]. Fuzzy Cognitive Maps have been used for decision analysis, and operation research [1]. The objective of this paper is to focus on the construction and the use of FCM in modeling systems. It will be shown that FCMs are useful to exploit the knowledge and experience that human have accumulated for years on the operation of a complex plant. Such methodologies are crude analogs of approaches that exist in human and animal systems and have their origins in behavioral phenomena related to these beings [10]. So, FCM represents knowledge in a symbolic manner and relates states, variables, events and inputs in an analogous to beings manner. This methodology can contribute to engineers’ intention to construct more intel-
Fuzzy Cognitive Maps (FCM) is a modeling methodology for complex systems, which originated from the combination of Fuzzy Logic and Neural Networks. The graphical illustration of an FCM is a signed fuzzy graph with feedback, consisting of nodes and weighted interconnections. Nodes of the graph stand for concepts that are used to describe main behavioral characteristics of the system. Nodes are connected by signed and weighted arcs representing the causal relationships that exist among concepts. With the graphical representation it is illustrated, which concept influences other concepts, showing the interconnections between concepts. This simple illustration permits thoughts and suggestions in reconstructing FCM, as the adding or deleting of an interconnection or a concept. In conclusion, an FCM is a fuzzy-graph structure, which allows systematic causal propagation, in particular forward and backward chaining.

2. Fuzzy Cognitive Maps

Fuzzy Cognitive Maps (FCM) is a modeling methodology for complex systems, which originated from the combination of Fuzzy Logic and Neural Networks. The graphical illustration of an FCM is a signed fuzzy graph with feedback, consisting of nodes and weighted interconnections. Nodes of the graph stand for concepts that are used to describe main behavioral characteristics of the system. Nodes are connected by
Fuzzy cognitive map can avoid many of the knowledge-extraction problems which are usually posed by rule-based systems. Fuzzy Cognitive Maps are used to represent and to model the knowledge on the examining system. Existing knowledge on the behavior of the system is embedded in the structure of nodes and interconnections of the FCM. Each node-concept represents one of the key-factors of the system. Relationships between concepts have three possible types; either express positive causality between two concepts ($W_{ij} > 0$) or negative causality ($W_{ij} < 0$) or no relationship ($W_{ij} = 0$). The value of $W_{ij}$ indicates how strongly concept $C_i$ influences concept $C_j$. The sign of $W_{ij}$ indicates whether the relationship between concepts $C_i$ and $C_j$ is direct or inverse. The direction of causality indicates whether concept $C_i$ causes concept $C_j$, or vice versa. These parameters have to be considered when a value is assigned to weight $W_{ij}$.

A new mathematical formulation for calculating the values of concepts of Fuzzy Cognitive Map, is proposed:

$$A_i^t = f\left(\sum_{j=1, j\neq i}^{n} A_{j}^{t-1} W_{ij} + k_2 A_i^{t-1}\right)$$  \hspace{1cm} (1)

The coefficient $k_2$ represents the proportion of the contribution of the previous value of the concept in the computation of the new value and the $k_1$ expresses the influence of the interconnected concepts in the configuration of the new value of the concept $A_i$. This new mathematical formulation assumes that a concept links to itself with a weight $W_{ii} = k_2$ [15].

In the following of this paper, it is assumed that the influence of the previous value of each concept is high and it is supposed that $k_1 = k_2 = 1$. This means that the previous value of each concept has a great influence in the determination of the new value. The inclusion of the previous value of each concept in the calculation rule results in smoother variation on the values of concepts after each recalculation of their value. This will become apparent on the example of Section 3.
So, Eq. (3) computes the new state vector $A'$, which results from the multiplication of the previous, at time $t - 1$, state vector $A^{t-1}$ by the weight matrix $W$. The new state vector holds the new values of the concepts after the interaction among concepts of the map and the adding of the state vector $A^{t-1}$.

But Eq. (3) can be written as:

$$A' = f(A^{t-1}W_{\text{new}}),$$

where, the new weight matrix $W_{\text{new}}$ is the weight matrix $W$ of the Fuzzy Cognitive Map with the entire diagonal elements equal to unit, which means that each concept causes itself with a weight $W_{ii} = 1$. This is a new approach, which differs from the existing representation of Fuzzy Cognitive Map in the literature, where it is assumed that no concept causes itself and so the diagonal of the weight matrix is zero.

2.2. Methods for construction of Fuzzy Cognitive Maps

The development and construction of Fuzzy Cognitive Map have great importance for its use in the modeling of systems. Fuzzy Cognitive Map represents the human knowledge on the operation of the system and experts develop FCMs using their experience and knowledge on the system. Construction methodologies rely on the exploitation of experts’ experience and knowledge on system’s model and behavior. Experts determine the number and kind of concepts that consist FCM and the interrelationships among concepts. Experts know the main factors that determine the behavior of the system, each one of these factors is represented by a concept. Experts according to their experience, they determine concepts of FCM that stand for events, actions, goals, values, and trends of the system. Experts know, which elements of the system influence other elements; for the corresponding concepts they determine the negative or positive effect of one concept on the others, with a fuzzy degree of causation. The determination of the degree of causal relationship among concepts can be improved by the application of learning rules for choosing appropriate weights for the FCM [8]. In this way, an expert decodes his own knowledge on the behavioral model of the system and transforms this knowledge in a weighted graph.

2.2.1. Assigning numerical weights

Knowledge on the behaviour of a system is rather subjective and in order to construct a model of the system it is proposed to utilize, the experience of a group of experts [13]. Experts are polled together and they determine the relevant factors that should stand as nodes of FCM. So, they decide the number of concepts, which consist the FCM and what characteristic of the system each concept represents. Then, experts are individually asked to express the causal relationship among these concepts. The result of this procedure will be a collection of individual FCMs, with the same nodes but different links among concepts or different weights of interconnections. The individual FCMs must be combined into one collective FCM and a method to combine the individual maps is needed. A first approach could be the summation of different weight matrix:

$$W = f\left(\sum_{1}^{N}W_k\right),$$

where $W$ is the overall matrix, $W_k$ is the individual weight matrix, which each one of the $N$ experts has developed, and $f$ is a threshold function, usually a type of the sigmoid function that will transform the sum of weights in the interval $[-1, 1]$.

It is common that experts have different experience and subjective knowledge on the system [4]. Thus it is considered that there are experts of different credibility on the knowledge of the system, and for these experts their contributions on constructing FCMs may be multiplied by a nonnegative ‘credibility’ weight $b_k$.

$$W = f\left(\sum_{1}^{N}b_kW_k\right),$$

where $b_k$ is the credibility weight for $k$-th expert and $W_k$ is the weight matrix of $k$-th expert’s Fuzzy Cognitive Map and $N$ is the number of the experts. But in this case, another mechanism must be used to determine who and how credibility weights will be assigned to every expert. As an example one expert could be “penalized” with an extremely low or zero credibility weight if the expert’s choice differs from other experts’ weight choice.
A new advanced algorithm is proposed in order to assign weights for each interconnection and credibility weights for experts. Every expert constructs an individual FCM. Then for each one interconnection of the overall FCM the corresponding weights from each individual map are collected together and compared according to the following algorithm. In this algorithm, the average value of the proposed weights for each interconnection is used.

First of all, the sign of the proposed weights are examined. If the number of weights with the same sign is less than $\pi N$ this means that it is not very clear among experts the positive or negative causality between two concepts and so they are asked to reassign weights. Otherwise, the procedure continues and the proposed weights are used to determine the average weight. Each expert that assign a weight for one interconnection not close enough to the average weight is penalized and the corresponding weight is partially taking into account. This mechanism is implemented using the following algorithm:

**Algorithm 1.**

**Step 1.** For all the $N$ experts, set credibility weight $b_k = 1$.

**Step 2.** For $i,j = 1$ to $n$

**Step 3.** For each interconnection ($C_i$ to $C_j$) examine the $N$ weights $W_{ij}^k$ that each $k$-th of the $N$ experts has assigned.

**Step 4.** If there are weights $W_{ij}^k$ with different sign and the number of weights with the same sign is less than $\pi N$

THEN

ask experts to reassign weights for this particular interconnection and go to step 3

ELSE

take into account the weights of the greater group with the same sign and consider that there are no other weights and penalize the experts who chose "wrong" signed weight with a new credibility weight $b_k = \mu_1 * b_k$.

**Step 5.** For the weights with the same sign, find their average value

$$W_{ij}^{ave} = \frac{\sum_{k=1}^{N} b_k W_{ij}^k}{N}.$$

**Step 6.** IF $|W_{ij}^{ave} - W_{ij}^k| \geq 0.2$

THEN

consider that there is no weight $W_{ij}^k$, penalize the $k$-th expert $b_k = \mu_2 * b_k$ and go to step 5

ELSE

construct the new weight matrix $W$ which has elements the weights $W_{ij}^{ave}$

**Step 7.** If there is no examined interconnection go to step 2

END.

**Example 2.1.** Six experts have constructed six individual FCMs. Experts have suggested the following weights for the interconnection from concept $C_i$ to concept $C_j$:

$W_{ij} = [-0.5, 0.6, 0.66, 0.7, 0.65, 0.25]$. For this example, the requested number of weights with the same sign is $\pi = 0.8$. Moreover $\omega_1 = 0.2$ and $\mu_1 = \mu_2 = 0.9$. According to step 3 of the algorithm, the majority of experts have assigned positive weights to that interconnection so the 1-st expert is penalized with credibility weight $b_1 = \mu_1 * b_1 = 0.9 b_1$ and the corresponding weight is dropping out. Then, at step 4 of the algorithm, the average weight is computing $W_{ij}^{ave} = 0.572$, and it is compared with the proposed weights, but according to step 6 of the algorithm, the weight suggested by the 6-th expert with value 0.25 is excluded from the calculation and the 6-th expert is penalized. The rest of the weights are used to calculate the new average weight. In this case, the chosen weight has the value $W_{ij}^{ave} = 0.652$ for this particular weight interconnection. The same methodology is used to assign weights for all the interconnections and construct the overall Fuzzy Cognitive Map.

2.2.2. Assigning linguistic variables for FCM weights

Another methodology to construct a Fuzzy Cognitive Map that is closer to fuzzy logic is proposed now. Experts are asked to describe the causality among concepts using linguistic notions. Every expert will determine the influence of one concept to the other as “negative” or “positive” and then he will describe the
grade of influence with a linguistic variable such as "strong", "weak" and etc [9].

Influence of one concept on another is interpreted as a linguistic variable taking values in the universe $U = [-1, 1]$ and its term set $T_{(influence)}$ could be:

$T_{(influence)} = \{\text{negatively very strong}, \text{negatively strong}, \text{negatively medium}, \text{negatively weak}, \text{zero}, \text{positively weak}, \text{positively medium}, \text{positively strong}, \text{positively very strong}\}$.  

The semantic rule $M$ is defined as follows and these terms are characterized by the fuzzy sets whose membership functions are shown in Fig. 2:

$M_{(negatively \ \text{very } strong)} = \text{the fuzzy set for \ "an influence below to } -75\%\text{" with membership function } \mu_{nvs}$.

$M_{(negatively \ \text{strong})} = \text{the fuzzy set for \ "an influence close to } -75\%\text{" with membership function } \mu_{ns}$.

$M_{(negatively \ \text{medium})} = \text{the fuzzy set for \ "an influence close to } -50\%\text{" with membership function } \mu_{nm}$.

$M_{(negatively \ \text{weak})} = \text{the fuzzy set for \ "an influence close to } -25\%\text{" with membership function } \mu_{nw}$.

$M_{(zero)} = \text{the fuzzy set for \ "an influence close to } 0\text{" with membership function } \mu_{z}$.

$M_{(positively \ \text{weak})} = \text{the fuzzy set for \ "an influence close to } 25\%\text{" with membership function } \mu_{pw}$.

$M_{(positively \ \text{medium})} = \text{the fuzzy set for \ "an influence close to } 50\%\text{" with membership function } \mu_{pm}$.

$M_{(positively \ \text{strong})} = \text{the fuzzy set for \ "an influence close to } 75\%\text{" with membership function } \mu_{ps}$.

$M_{(positively \ \text{very strong})} = \text{the fuzzy set for \ "an influence above to } 75\%\text{" with membership function } \mu_{pvs}$.

The linguistic variables that describe each interconnection are combined and the overall linguistic variable is transformed in the interval $[-1,1]$. A numerical weight for each interconnection will be the outcome of the defuzzifier, where the Center of Gravity method is used to produce this crisp weight [11]. This methodology has the advantage that experts do not have to assign numerical causality weights but to describe the degree of causality among concepts.

2.2.3. Synthesizing different Fuzzy Cognitive Maps

It is supposed a distributed system, for each subsystem a distinct FCM is constructed and then all FCMs can be combined in one augmented Fuzzy Cognitive Map with weight matrix $W$ for the whole system. The unification of the distinct FCM depends on the concepts of the segmental FCM, if there are no common concepts among different maps, the combined matrix $W$ is constructed according to Eq. (7). In this case, there are $K$ different FCM matrices, with weight matrices $W_j$ and the dimension of matrix $W$ is $n \times n$ where $n$ equals the total number of distinct concepts of all the FCMs.
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\[ W = \begin{bmatrix} W_1 & 0 & \cdots & 0 \\ 0 & W_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & W_K \end{bmatrix} \]  \hspace{1cm} (7)

**Example 2.2.** It is assumed that there are two Fuzzy Cognitive Maps, \( F_1 \) with concepts \( C_1, C_2, C_3 \) and \( F_2 \) with concepts \( C_4, C_5, C_6 \). Weight matrices for \( F_1 \) and \( F_2 \) are:

\[ W_1 = \begin{bmatrix} 0 & 0 & W_{13} \\ W_{21} & 0 & 0 \\ W_{31} & W_{32} & 0 \end{bmatrix} \]

and

\[ W_2 = \begin{bmatrix} 0 & W_{45} & W_{46} \\ W_{54} & 0 & W_{56} \\ 0 & W_{65} & 0 \end{bmatrix} \]

The augmented weight matrix will be:

\[ W = \begin{bmatrix} W_1 & 0 \\ 0 & W_2 \end{bmatrix} \]

\[ = \begin{bmatrix} 0 & 0 & W_{13} & 0 & 0 & 0 \\ W_{21} & 0 & 0 & 0 & 0 & 0 \\ W_{31} & W_{32} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & W_{45} & W_{46} & 0 \\ 0 & 0 & W_{54} & 0 & W_{56} & 0 \\ 0 & 0 & 0 & 0 & W_{65} & 0 \end{bmatrix} \]

But, in most cases, the unification is used because there are common concepts among the distinct FCMs and our intention is the construction of an enhanced Fuzzy Cognitive Map. In this case, there will be an overlapping between some of the diagonal elements-matrices of matrix \( W \) in Eq. (7). Overlapping represent weights of interconnections between concepts that belong to different FCMs. Then, segmental FCMs with common concepts, are combined together, calculating new weights for the interconnection between common concepts. If there are more than one common concept between the Fuzzy Cognitive Maps, there will be proposed two or more weights for the same interconnection. In this case, as new weight will be the average of weights. Then, Eq. (7) is implemented to construct the weight matrix of the overall Fuzzy Cognitive Map. It is consisted of \( n \) concepts that correspond to the total number of the different concepts that there are at all the segmental FCMs.

**Example 2.3.** It is assumed that there are two Fuzzy Cognitive Maps, \( F_1 \) with concepts \( C_1, C_2, C_3 \) and \( F_2 \) with concepts \( C_2, C_3, C_4, C_5 \). Weight matrices for \( F_1 \) and \( F_2 \) are:

\[ W_1 = \begin{bmatrix} 0 & 0 & W_{13} \\ W_{21} & 0 & 0 \\ W_{31} & W_{32} & 0 \end{bmatrix} \]

and

\[ W_2 = \begin{bmatrix} 0 & W_{23} & W_{24} & 0 \\ W_{32} & 0 & W_{34} & W_{35} \\ W_{42} & W_{43} & 0 & 0 \\ W_{52} & W_{53} & W_{54} & 0 \end{bmatrix} \]

The augmented Fuzzy Cognitive Map will have five concepts and its weight matrix will be:

\[ W = \begin{bmatrix} 0 & 0 & W_{13} & 0 & 0 \\ W_{21} & 0 & W_{23} & W_{24} & 0 \\ W_{31} & W_{32} & 0 & W_{34} & W_{35} \\ 0 & W_{42} & W_{43} & 0 & 0 \\ 0 & W_{52} & W_{53} & W_{54} & 0 \end{bmatrix} \]

2.3. **Neural network nature of Fuzzy Cognitive Maps**

Fuzzy Cognitive Maps have been described as a hybrid methodology, because it combines characteristics of fuzzy logic and neural networks. The development and construction of FCMs have shown their fuzzy nature. Well-known learning rules from Neural
Networks theory, they are used to train the Fuzzy Cognitive Map. Parameter learning of FCM concerns the updating of interconnection weights among concepts.

The construction of FCM is based on experts who determine concepts and weighted interconnections among concepts. This methodology may lead to a distorted model of the system because human factor is not always reliable. In order to refine the model of the system, learning rules are used to adjust weights of FCM interconnections. The Differential Hebbian learning rule has been proposed [8] to be used in the training of a specific type of FCMs. The Differential Hebbian learning law adjusts the weights of the interconnection between concepts. It grows a positive edge between two concepts if their values increase or decrease and it grows a negative edge if values of concepts move in opposite directions. Adjusting the idea of differential Hebbian learning rule in the framework of Fuzzy Cognitive Map, the following rule is proposed to calculate the derivative of the weight between two concepts:

\[ w'_{ji} = -w_{ji} + s(A_{j}^{\text{new}})s(A_{i}^{\text{old}}) \]

\[ + s'(A_{j}^{\text{new}})s'(A_{i}^{\text{old}}), \quad (8) \]

where

\[ S(x) = \frac{1}{1 + e^{-kx}}. \]

New learning rules for Fuzzy Cognitive Maps need more investigation. These rules will give FCMs useful characteristics such as the ability to learn arbitrary non-linear mappings, capability to generalize to situations, the adaptivity and the fault tolerance capability [5].

3. Fuzzy Cognitive Map for direct control of a process

Fuzzy Cognitive Map can be used to model and to control a process [3]. Such an application of Fuzzy Cognitive Map will be examined. The existing knowledge on the operation and behavior of the system is represented by an FCM. It models the operation and best describes the behavior of the process. Generally, the need of using Fuzzy Cognitive Map is motivated if the nature of the process is such that analytic models do not exist or are inadequate, but human operator can manually control the process to a satisfactory degree.

An important issue in development a Fuzzy Cognitive Map is the determination of the concepts that best describe the system and the direction and grade of causality among concepts. This example will reveal how a Fuzzy Cognitive Map is constructed, how concepts are chosen, how values are assigned to the interconnections between concepts. Then, the constructed Fuzzy Cognitive Map is used to model and to control the process. The operation of the system is simulated and simulation results are discussed.

The considered system consists of one tank and three valves that influence the amount of liquid in the tank; Fig. 3 shows an illustration of the system. Valve1 and valve2 empty two different kinds of liquid into tank1 and during the mixing of the two liquids a chemical reactions takes place into the tank. A sensor is located inside the tank and it measures the specific gravity of the liquid that is produced into tank, by the mixing of the two incoming liquids. When value of specific gravity lies in the range between \( G_{\text{max}} \) and \( G_{\text{min}} \), this means that the desired liquid has been produced in tank. Moreover, there is a limit on the height of liquid in tank, which cannot exceed an upper limit \( H_{\text{max}} \) and a lower limit \( H_{\text{min}} \). So the control target is to keep these variables in the range of values:

\[ G_{\text{min}} \leq G \leq G_{\text{max}}, \]

\[ H_{\text{min}} \leq H \leq H_{\text{max}}. \quad (9) \]

The first step in constructing the Fuzzy Cognitive Map, which will model and control this simple system, is the determination of the concepts of the FCM. Concepts will stand for the variables and states of the process as the height of liquid in the tank or the state of valves. So a primitive FCM is developed with five concepts and later any new concept, which can improve the model and control of the system, can be added.

Concept1. The amount (height) of liquid that tank contains. This amount is dependent on the state of valve1, valve2 and valve3.
Concept 2. The state of the valve 1. Valve is closed, open or partially opened.

Concept 3. The state of the valve 2. Valve is closed, open or partially opened.

Concept 4. The state of the valve 3. Valve is closed, open or partially opened.

Concept 5. The specific gravity of the produced liquid in the tank.

After having selected the concepts that can represent the model of the system and its operating behavior, the interconnections between concepts must be determined. At first, experts decide for each concept with which other concept it will be connected and they determine the sign and weight of each connection.

The connections between concepts are:

Event 1. It connects concept 2 (valve 1) with concept 1 (amount of liquid in the tank). It relates the state of the valve 1 with the amount of the liquid in tank.

Event 2. It relates concept 3 (valve 2) with concept 1 (amount of liquid in the tank); valve 2 causes the increase or not of the amount of liquid in tank.

Event 3. It connects concept 4 (valve 3) with concept 1; the state of valve 3 causes the decrease or not of the amount of liquid in tank.

Event 4. It relates concept 1 (height of liquid) with concept 2; when the height of the liquid in tank is high, valve 1 (concept 2) needs closing and so the amount of incoming liquid into tank is reduced.

Event 5. It connects concept 1 (height of liquid in tank) with concept 3 (state of valve 2); when the height of the liquid in tank is high, the closing of valve 2 (concept 3) reduces the amount of incoming liquid.

Event 6. It connects concept 5 (the specific gravity of the liquid) with concept 4 (valve 3). When the quality of the liquid in the tank is the desirable, valve 3 is opened and the produced liquid continues to another tank-process.

Event 7. It shows the effect of concept 1 (amount of liquid in tank) into concept 5 (specific gravity of the liquid). When the amount of liquid in tank changes, this influences the specific gravity of the liquid.

Event 8. It relates concept 5 (specific gravity of liquid) with concept 2 (state of valve 1), when the specific gravity is very low then valve 1 (concept 2) is opened and liquid comes into tank.

Figure 4 shows the graphical representation of the Fuzzy Cognitive Map that is used to describe, model and control this simple system. The initial value of each concept, the interconnections and the weights among concepts are illustrated. The values of con-
cepts correspond to the physical measurement of their physical magnitude. Each concept has a value, which ranges between \([0,1]\) and it is obtained after thresholding the real value of the concept. It is apparent that an interface is needed, which will transform the physical measures of the system to their representative values in the FCM mode and vice versa. Initial measurements of the real system have transformed to concept values and the initial vector is formed:

\[
A_0 = \begin{bmatrix} 0.1 & 0.45 & 0.39 & 0.04 & 0.01 \end{bmatrix}.
\]

For this system, three experts have been used in order to construct the Fuzzy Cognitive Map. They jointly determined the concepts of the FCM and then each expert drew the interconnections among concepts and he assigned a weight for each interconnection. Then, algorithm of Section 2.2.1 was applied and the following weight matrix was produced:

\[
W = \begin{bmatrix}
0 & -0.4 & -0.25 & 0 & 0.3 \\
0.36 & 0 & 0 & 0 & 0 \\
0.45 & 0 & 0 & 0 & 0 \\
-0.9 & 0 & 0 & 0 & 0 \\
0 & 0.6 & 0 & 0.3 & 0
\end{bmatrix}.
\]

For these initial values of concepts, Fuzzy Cognitive Map starts to simulate the behavior of the process. At each step of the simulation values of concepts are calculated according to Eq. (2). As simulation step of the FCM is defined the time step during which the values of the concepts are calculated and change. The value of each concept is determined by the result of taking all the causal weights pointing into this concept and multiplying them by the value of the corresponding concept and adding the previous value of each concept. Then, a threshold function is used and so the result belongs to the range \([0, 1]\). In this example, the following sigmoid function is used:

\[
f(x) = \frac{1}{1 + e^{-x}}.
\]

The FCM program was written and simulations run in Matlab 5.0 by Mathworks, 1997. Simulation results for the values of concepts for ten steps are depicted in Table 1. The variation of concepts’ values is appeared at Fig. 5. Examining Fig. 5, it can be seen that Fuzzy Cognitive Map converges to an equilibrium region after seven simulation steps, which is a satisfactory result.

These simulation steps are independent to the real system, when FCM reaches an equilibrium region new values of concepts will be transformed to the corresponding real values and vice versa. In this example, when FCM reaches the equilibrium region, values of concept C2, C3, C4 are transmitted to the real system and cause the opening of valves to 70.7% for valve1, 61.2% for valve2 and 71.1% for valve3. Then, measurements of height of liquid and the specific gravity

![Fig. 4. The FCM model of the process example with initial values.](image)
of liquid are transmitted to Fuzzy Cognitive Map. New measurements on the real system are transformed to the following values of concepts: concept C1=0.4 and concept C5=0.3 so the new vector will be:

\[ A_0 = [0.4 \quad 0.7077 \quad 0.6120 \quad 0.7171 \quad 0.3]. \]

Then FCM interacts again. The results of eight steps are depicted on Table 2 and the variation of the values of concepts on Fig. 6. This mechanism of controlling the process will continue. FCM-controller will receive the measured values of the real system, it will interact, it will reach the desirable equilibrium region and it will transmit the values of concepts to the real system and so on.

4. Features of FCM for supervisory control systems problems

Complex systems are characterized with high dimension and their dynamics are unknown but conventional techniques cannot easily handle this kind of systems. The application of Fuzzy Cognitive Maps for the modeling of the supervisor of the complex system seems to be a prospective approach. The hierarchical structure of Fig. 7 is proposed to model large scale complex systems. At the lower level of the structure will lie the plant, which is controlled through conventional controllers. These controllers perform the usual tasks and reflect the model of the plant during normal operation conditions. The supervisor of the system is modeled as a Fuzzy Cognitive Map.
There is an amount of information that must pass from the lower level to the Supervisor-FCM. So an interface is needed, which will filter, transform and pass information from the local controllers to the FCM on the upper level. Then Fuzzy Cognitive Map will interact using Eq. (2); concepts of FCM will have new values that must be transmitted to the conventional controllers. So, the interface will follow the opposite direction. In this way changes on one or more concepts of the FCM could mean change in the value of one or more elements of the system.

The model of Fuzzy Cognitive Map can be expanded to include advanced features, such as fault diagnosis, effect analysis [12] or planning and decision making characteristics. Some of the concepts of the FCM could stand for device failure modes, their effects and causes, a subsystem’s normal or irregular operation, the functionality of the system, the failures, the system mission, and the ultimate function of the system. So the FCM would represent the failure modes and their effects and the relations among them, that an expert uses to describe the functionality of the system and failures.

A very interesting quality of FCM is their ability in prediction and redesigning of the system. This can help the designer in evaluating what would happen if some parameters of the system change. Another useful characteristic of the FCM is its efficiency in prediction and especially to predict what would be the result of a scenario or what will be the consequences for the whole process if a state changes suddenly. This feature is especially useful for designers of systems to observe the influence of each device separately.

With Fuzzy Cognitive Maps the knowledge and human operator experience is exploited. The human coordinator of a system knows the operation of a whole

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**Fig. 6. Variation of values of 6 concepts for the FCM of the process for eight simulation steps.**

---

**Table 2**

<table>
<thead>
<tr>
<th>Step</th>
<th>Tank1</th>
<th>Valve1</th>
<th>Valve2</th>
<th>Valve3</th>
<th>Specific gravity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.4000</td>
<td>0.7077</td>
<td>0.6120</td>
<td>0.7171</td>
<td>0.3000</td>
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<td>0.6120</td>
<td>0.7171</td>
<td>0.7105</td>
</tr>
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</table>
system and uses a mental model consisted of concepts to describe it. He relates the operation of one sub-system or two different subsystems to a concept, or a concept stand for a specific procedure.

FCM models the supervisor and it is consisted of concepts that may represent the irregular operation of some elements of the system, failure mode variables, failure effects variables, failure cause variables, severity of the effect or design variables. Moreover, this FCM will include concepts for determination of a specific operation of the system and it will be used for strategic planning and decision analysis. The supervisor FCM will represent vital components of the plant and will reflect the operational state of the plant. The development of this FCM requires the integration of several expert opinions in order to construct a FCM with diagnosis and predictive capabilities.

Example 4.1. The supervisor for a complex process will be developed. Four experts working on a chemical industry, which produce refreshments from water, sugar, fruit juice etc, were asked to develop a Fuzzy Cognitive Map. This FCM will be used as a supervisor of the whole plant, which will describe the operation of a process, the final product of the process and the different aspects that determine the quality of the product. Experts developed the Fuzzy Cognitive Map, which is depicted on Fig. 8. They decided that the most important concept is the quality of the produced product. They developed an FCM around the main concept C1, which represents the “product degradation” of the final product. Then, experts determined the other concepts of the real system that influence this concept, so concept C1 depends on:

- Concept C2 “the internal variation of the process”,
- Concept C3 “the poor quality of the input material”,
- Concept C4 “wear and tear machine parts”,
- Concept C5 “technical malfunction”,
- Concept C6 “poor operator settings”,
- Concept C8 “machine shut down”.

Other concepts that consist the Fuzzy Cognitive Map are:

- Concept C7 “reschedule the process”,
- Concept C9 “maintenance”.

Then, the interrelations among concepts were determined with the following logical procedure. The value of concept 1 “degradation of product” increases the need to “reschedule the process” which is presented as concept C7.
Concept C7 decreases the value of concepts C6 “poor operator setting” and concept C2 “Internal process variation”.

Concept C4, which stand for “wear and tear machine parts”, has a positive influences on concept C5 “technical malfunction”.

Concept C5 “technical malfunction” increase the amount of concept C9 “the maintenance” and the amount of concept C8 “the machine shut down”.

Concept C9 “maintenance” decreases the amount of the following concepts: concept C5 “technical malfunction”, concept C8 “the machine shut down” and concept C4 “wear and tear machine parts”.

Concept C8 “machine shut down” increases the amount of concept C7 “reschedule process” and increases the value of concept C9 “maintenance”.

Then, experts were asked to assign values on the interconnections among concepts. Four FCMs were constructed with the same concepts, but with 4 different weights on each interconnection. Then algorithm of Section 2.2.1 was implemented and an augmented Fuzzy Cognitive Map was constructed, which is depicted on Fig. 9 and it is used to supervise the plant.

For the constructed Fuzzy Cognitive Map, values were assigned to the concepts and the simulation of the FCM was started. Equation (2) is used to calculate the new values of concepts after each step of the FCM. Table 3 gathers the initial values of concepts and their values for five simulation steps. FCM reaches an equilibrium region and if a new value for one or more concepts come from the lower level then after a limited number of steps, FCM will reach the equilibrium region.

The development of the supervisor-FCM that is dedicated to a particular plant depends on the supervisory-coordinator tasks that the user of overall system requires. A complete Fuzzy Cognitive Map would include a decision making part and a planning part.

Table 3
The values of concepts for the supervisor-FCM for 5 steps

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<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>C8</th>
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5. Summary

Fuzzy Cognitive Map Theory, a new soft computing approach is used to model the behavior of complex systems. This technique best utilizes existing experience in the operation of the system and it has been examined. For such systems it is extremely difficult to describe the entire system by a precise mathematical model. Thus, it is more simple and useful to divide the whole plant in virtual parts and to construct an FCM for each part. The experience of different experts who can easily judge the variables and states of a small process and then unify these to construct the final system by integrating the different Fuzzy Cognitive Maps into augmented ones have been utilized. FCM approach represents systems in a graphical way showing the causal relationships between states-concepts and accomplishes the unification of knowledge by superposing small subsystems. FCMs offer the opportunity to produce better knowledge based system applications, addressing the need to handle uncertainties and inaccuracies associated with real world problems.

An example of FCM model for supervisory control was developed. Supervisor is consisted of concepts that represent the irregular operation of some elements of the system, failure mode variables, failure effects variables, failure cause variables, severity of the effect or design variables. In the development of this FCM the integration of several expert opinions was included in order to achieve its diagnosis and predictive task and the results after simulation were shown. Methods for development more integrated supervisory models.

FCMs are a type of symbolic methodology, which can increase the effectiveness, autonomy and intelligence of systems. Since this symbolic method on modeling and controlling a system is easily adaptable and relies on human expert experience and knowledge, it can in a sense be considered intelligent.

References


